

ADDENDUM

Image Color Calibration: Extinction Coefficients and Weight Factors

Stephen J. Maas

Color calibrating the raw imagery from your astro-imaging camera allows you to produce processed images that display a realistic representation of the imaged object without subjective manipulation with Photoshop. This works for both monochrome and color cameras. With a monochrome camera, we use color filters to acquire images in the red, green and blue spectral bands. With a color camera, the raw color image can be “de-Bayered” into separate red, green and blue images which can then be handled the same way as images from a monochrome camera. In either case, the red, green and blue images need to be calibrated so that, when they are combined, the resulting color image has the correct color balance.

Color calibration can be explained by a simple equation:

$$S'_{i,j} = S_{i,j} / (A_j W_j) \quad [\text{Eq. 1}]$$

in which S is the raw “signal” produced by the camera and S' is the calibrated signal. The subscript i indicates an individual pixel in the image, while the subscript j indicates the spectral band of the image. The signal could be in terms of accumulated photons, but the output of most digital cameras are “analog-to-digital units” (ADUs), a dimensionless scaled version of the photon flux.

The terms in the denominator facilitate calibration of the image. A_j is the atmospheric transmissivity factor, while W_j is a weight factor sometimes called the “white-balance”. Their values are specific to a spectral band. A_j can be calculated as follows:

$$A_j = 10^{-0.4 k_j [X-1]} \quad [\text{Eq. 2}]$$

Here, X is the “optical thickness” and is a function of the zenith angle Z (the angle between the object and the zenith):

$$X = \sec(Z) - 0.0018[\sec(Z)-1] - 0.0029[\sec(Z)-1]^2 - 0.0008[\sec(Z)-1]^3 \quad [\text{Eq. 3}]$$

When the object is directly overhead ($Z = 0$), $A_j = 1$. As the object approaches the horizon, Z increases and the resulting value of A_j decreases. Since A_j is in the denominator of Eq. 1, decreasing its value compensates for the corresponding decrease in the value of the raw signal ($S_{i,j}$) due to increased atmospheric scattering and absorption along a longer optical path through the atmosphere. The rate at which A_j decreases as a function of X is controlled by the extinction coefficient k_j , the value of which is also affected by the overall optical clarity of the atmosphere. When the atmosphere is clearer, the value of k_j is smaller so that the brightness of an object decreases more slowly as it approaches the horizon. Since the atmosphere affects the various

wavelengths of light differently, the value of k_j is different for the red, green and blue spectral bands.

Note that A_j is not the actual transmissivity of the atmosphere but rather the ratio of the transmissivity at the zenith angle of the object to the transmissivity at $Z = 0$. Thus, calibrating the raw camera signal using A_j normalizes the effects of atmospheric transmissivity for images acquired at different zenith angles to make it seem like they were all acquired with the object directly overhead.

The value of W_j is related to the characteristics of your imaging equipment (camera, filters). W_j is a relative measure that expresses how the performance of your equipment in the three spectral bands must be adjusted to achieve “white balance” (white balance will be described later in this document). It’s defined so that the greatest value of W_j among the three spectral bands is set equal to 1, with the values for the two remaining bands being fractions of 1. Since W_j is in the denominator of Eq. 1, the fractional values act to increase the corresponding calibrated signal values (S'_{ij}) to achieve balance among the three spectral bands. Because your choice of camera and filters is probably different from other people’s, the values of W_j will be unique to your imaging setup.

Depending on your location and climate, k_j can exhibit substantial seasonal and day-to-day variations. While the values of W_j can probably be considered constants for your setup, you still must evaluate them. To achieve *objective* color calibration of your imagery, you should ignore the default values for k_j and W_j that you often find in image processing software packages and supply your own values that represent the atmospheric conditions when you acquired your imagery and the characteristics of your own imaging equipment. In the remainder of this document, I’ll describe how to use astro-imagery to determine your own values for k_j and W_j .

Evaluating k_j

To evaluate k_j , you’ll need to image one or more stars for several hours as they move from a position high in the sky to down near the horizon. You can use stars of *any spectral class* for this because atmospheric transmissivity affects the light of all stars the same way. I’ll use an example involving four stars that I imaged in 2015. The imaging session started at 9:11 PM on 11 March and ended at 12:49 AM on 12 March. Characteristics of the stars are summarized in **Table 1**.

Table 1. Characteristics of the four stars used in the example.

| # | Star ID | RA (h:m:s) | Dec (d:m:s) | Magnitude |
|---|--------------------|------------|-------------|-----------|
| 1 | HIP19767 (HD26749) | 04:14:18 | +12:20:53 | 6.74 |
| 2 | HIP22349 (HD30455) | 04:48:42 | +18:42:34 | 6.95 |
| 3 | HIP20577 (HD27859) | 04:24:29 | +16:53:10 | 7.79 |
| 4 | HIP20899 (HD28344) | 04:28:48 | +17:17:08 | 7.83 |

I used a QSI Model 583 monochrome camera with Baader RGB filters mounted on an Orion 120mm EON apochromatic refractor. I acquired images of the stars every hour. Each acquisition consisted of a set of 5 separate images of each star in the red, green and blue spectral bands.

Figure 1 shows the elevation and azimuth of the stars when images were acquired. Stars #3 and

#4 were close, so their plotted positions overlap. I had intended to acquire 4 sets of images for each star but stars #3 and #4 had sunk behind a tree by the time of the fourth acquisition. I ended up with 210 images. For this analysis, you don't have to start with stars directly overhead. Starting at around 20-30 degrees from the zenith is okay. It is important to image the stars down to as close to the horizon as possible, since it's near the horizon that A_j changes the most.

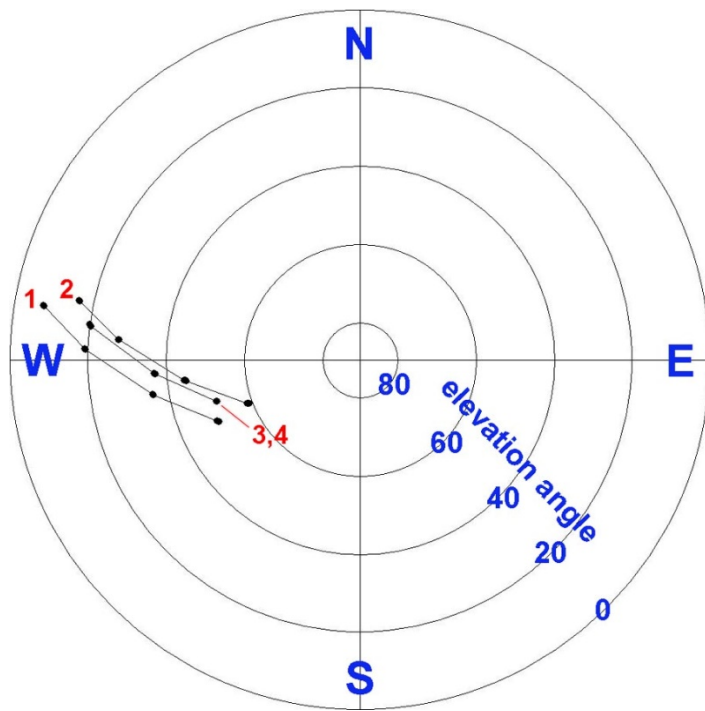


Figure 1. Elevation and azimuth positions (black dots) of the four target stars when images were acquired. Red numbers are the star numbers in Table 1.

Prior to acquiring your first set of images, you'll want to take some test images to determine an exposure time that results in the target stars being bright but *not saturated*. You must use the same exposure time for a given star over the entire imaging session. For this example, I used 4 seconds for stars #1 and #2, and 5 seconds for stars #3 and #4.

After acquiring the images, you need to extract image data for each target star. The first step is to open your images in image processing software and use a photometry tool to collect photon flux data. I used AIP4Win (<https://www.aavso.org/aip4win-no-longer-requires-registration>) but the process should be similar with other software packages. **Figure 2** shows a portion of a raw image along with the panel for AIP4Win's "Single Star Photometry" tool. Stellar photometry tools measure the photon flux from the star and then compare it to the photon flux from the starless region around it. This is done using an "aperture" centered on the star. As shown in Figure 2, this aperture is defined by three concentric rings. You specify the radii (in pixels) of these rings under the "Settings" tab. The inner ring should contain the star while the two outer rings define a donut-shaped annulus that measures the photon flux from the sky background. The tool will

subtract the background measurement from the star measurement and display the result under the “Result” tab. This value (60300.4) represents the raw “signal” ($S_{i,j}$) expressed in ADUs.

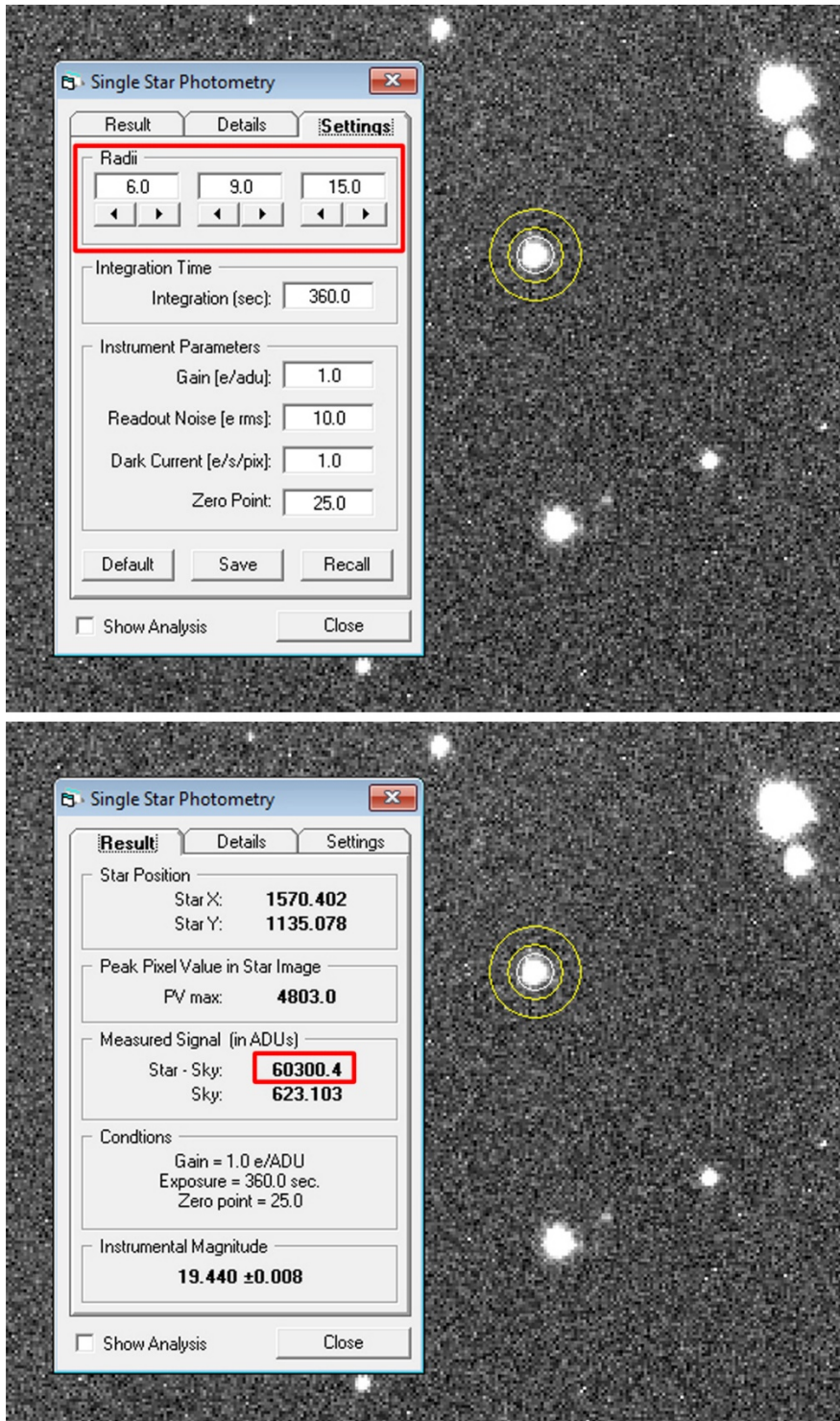


Figure 2. Portion of a raw image with AIP4Win’s Single Star Photometry tool.

One great thing about this approach is you can do it on a moonlit night. The photometer tool will remove the sky background brightness due to the moon. Also, *you don't need to apply dark frames to your images*. Since the effect of the sensor dark current is present in both the measurements for the star and the background, the photometer tool will automatically remove it. If you keep the target stars near the center of your images, you can skip applying flat frames too.

Once raw signal values have been determined for the target stars, they can be entered into spreadsheet software like Excel. **Figure 3** shows the spreadsheet for Star #1 in Table 1. Columns A, B and C contain the raw signal values for the red, green and blue spectral bands, respectively. There are four groups of five values in each column corresponding to the five images of the star taken at each of the four acquisition times.

| | A | B | C | D | E | F | G | H | I |
|----|--------|--------|--------|---------|---------|---------|------|------|---------|
| 1 | 534336 | 544510 | 457990 | 13.1888 | 13.2076 | 13.0346 | 50.5 | 39.5 | 1.29479 |
| 2 | 538209 | 533767 | 447599 | 13.196 | 13.1877 | 13.0116 | 50.3 | 39.7 | 1.29851 |
| 3 | 559365 | 554189 | 439467 | 13.2346 | 13.2253 | 12.9933 | 50.3 | 39.7 | 1.29851 |
| 4 | 529066 | 539140 | 436843 | 13.1789 | 13.1977 | 12.9873 | 50.3 | 39.7 | 1.29851 |
| 5 | 549022 | 537832 | 438226 | 13.2159 | 13.1953 | 12.9905 | 50.1 | 39.9 | 1.30228 |
| 6 | | | | | | | | | |
| 7 | 542977 | 515735 | 376532 | 13.2048 | 13.1534 | 12.8388 | 36.1 | 53.9 | 1.69319 |
| 8 | 588437 | 531985 | 376789 | 13.2852 | 13.1844 | 12.8394 | 36.1 | 53.9 | 1.69319 |
| 9 | 581611 | 520515 | 399445 | 13.2736 | 13.1626 | 12.8978 | 36.1 | 53.9 | 1.69319 |
| 10 | 551378 | 534653 | 385578 | 13.2202 | 13.1894 | 12.8625 | 35.9 | 54.1 | 1.70129 |
| 11 | 545014 | 523880 | 388949 | 13.2086 | 13.169 | 12.8712 | 35.9 | 54.1 | 1.70129 |
| 12 | | | | | | | | | |
| 13 | 509558 | 412897 | 269315 | 13.1413 | 12.9309 | 12.5036 | 19.4 | 70.6 | 2.98354 |
| 14 | 506493 | 407052 | 262287 | 13.1353 | 12.9167 | 12.4772 | 19.4 | 70.6 | 2.98354 |
| 15 | 504046 | 430772 | 248916 | 13.1304 | 12.9733 | 12.4249 | 19.2 | 70.8 | 3.01287 |
| 16 | 491727 | 409352 | 249014 | 13.1057 | 12.9223 | 12.4253 | 19.2 | 70.8 | 3.01287 |
| 17 | 498113 | 408281 | 257146 | 13.1186 | 12.9197 | 12.4574 | 19.2 | 70.8 | 3.01287 |
| 18 | | | | | | | | | |
| 19 | 256358 | 190189 | 67848 | 12.4543 | 12.1558 | 11.125 | 7.7 | 82.3 | 7.07903 |
| 20 | 253775 | 199669 | 64059 | 12.4442 | 12.2044 | 11.0676 | 7.7 | 82.3 | 7.07903 |
| 21 | 264475 | 182381 | 68973 | 12.4855 | 12.1138 | 11.1415 | 7.5 | 82.5 | 7.24676 |
| 22 | 295712 | 194778 | 71929 | 12.5971 | 12.1796 | 11.1834 | 7.5 | 82.5 | 7.24676 |
| 23 | 269740 | 184458 | 59546 | 12.5052 | 12.1252 | 10.9945 | 7.5 | 82.5 | 7.24676 |
| 24 | | | | | | | | | |

Figure 3. Excel spreadsheet with data for Star #1.

We now need to calculate the natural logarithm (base e) of the values in columns A, B and C using the mathematical functions in Excel. Stellar magnitudes are expressed on a logarithmic scale, so these calculations convert the raw signal values into “magnitudes”. These $\text{Ln}(S_{i,j})$ values are not actual visual magnitudes like in Table 1, but we don't need visual magnitudes for evaluating k_j . The calculated values of $\text{Ln}(S_{i,j})$ are shown in columns D, E and F of the spreadsheet for the red, green and blue spectral bands, respectively.

We have half the data that we need for determining k_j . We now need the zenith angle Z for the target star in each acquired image. If your telescope mount is connected to your imaging computer, the elevation angle of the scope might automatically be written to the header of each image file where it can be extracted using processing software. Alternately, you can use planetarium software like Cartes du Ciel (<https://www.ap-i.net/skychart/en/start>) or Stellarium

(<https://stellarium-web.org/>) to display the sky at the time of each image acquisition. Finding the target star and clicking on it should show information about the star, including its elevation.

Since I acquired the red, green and blue images of the target star in quick succession, I used a single elevation angle for all three images. Elevation angles (in degrees) for Star #1 are shown in column G of the spreadsheet. Since Z is the complement of the elevation angle, you can subtract the elevation angle from 90 degrees to get it. Zenith angles for Star #1 are shown in column H of the spreadsheet. Once we have Z , we can compute the optical thickness X using Eq. 3. Values of X for Star #1 are shown in column I of the spreadsheet.

If you use the charting function in Excel to plot the values of $\text{Ln}(S_{i,j})$ from columns D, E and F versus the corresponding values of X from column I, you should get something like **Figure 4**. I've let Excel add trend lines to the data points and used the "Display Equation on chart" option. These lines reveal that *the magnitude of a star decreases linearly as the optical thickness increases*.

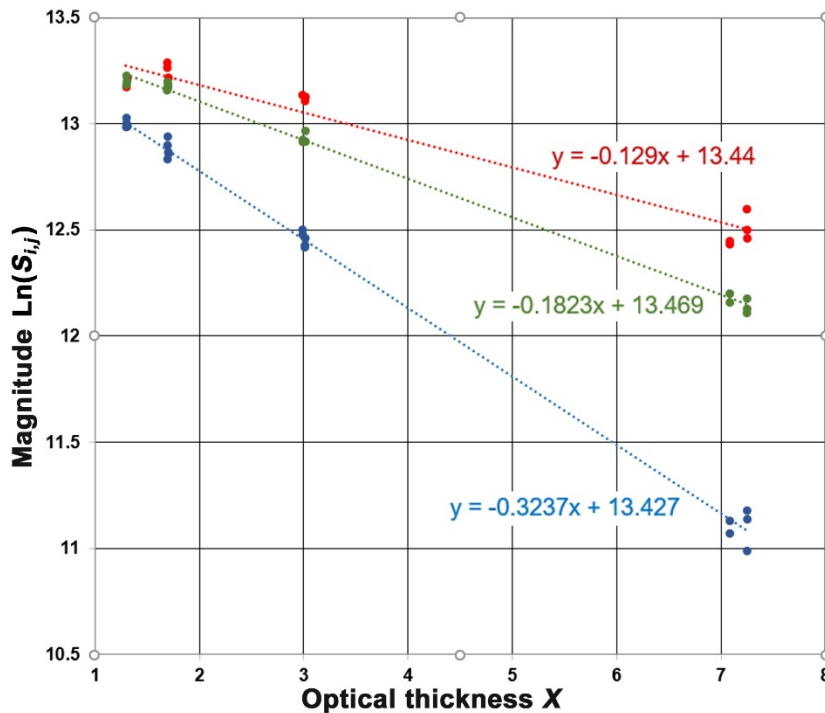


Figure 4. Graph of magnitude $\text{Ln}(S_{i,j})$ versus optical thickness X for Star #1.

More important is that *the values of the extinction coefficients are numerically equal to the slopes of the trend lines*. So, for Star #1, $k_r = 0.1290$, $k_g = 0.1823$, and $k_b = 0.3237$ (where the r , g and b subscripts denote the red, green and blue spectral bands). Since there's always random scatter in real data, averaging the results from several stars will produce more robust estimates. Averaging the results for the four stars in Table 1 yielded $k_r = 0.1380$, $k_g = 0.2050$, and $k_b = 0.3313$. Note that $k_b > k_g > k_r$. This should always be the case, since shorter wavelengths of light are affected more by atmospheric transmissivity than longer wavelengths.

Evaluating W_j

To evaluate W_j , we'll do what photographers normally do to calibrate the color of their photos: use a procedure called "white balancing". This is based on the observation that, when the sky is clear and the sun is high overhead, sunlight falling on a surface that equally reflects all wavelengths of visible light causes the surface to look white. So, in regular photography, I could expose my DSLR to sunlight falling on such a surface (a "calibration card") and adjust the camera's "white balance" so the card appears white in photos. If the card appears white, I can assume the colors of all other objects in the photos are correct.

The sun is a spectral class G2V star and there are quite a few G2V stars scattered across the sky (e.g., http://cat-star.org/dashboard/G2V/page_g2v.html). We can use them like calibration cards to white-balance our images. Specifically, we can determine values of W_j that, when applied in our image processing, cause unsaturated G2V stars in our images to appear white. When the G2V stars appear white, the colors of all the other objects in the images should be correct.

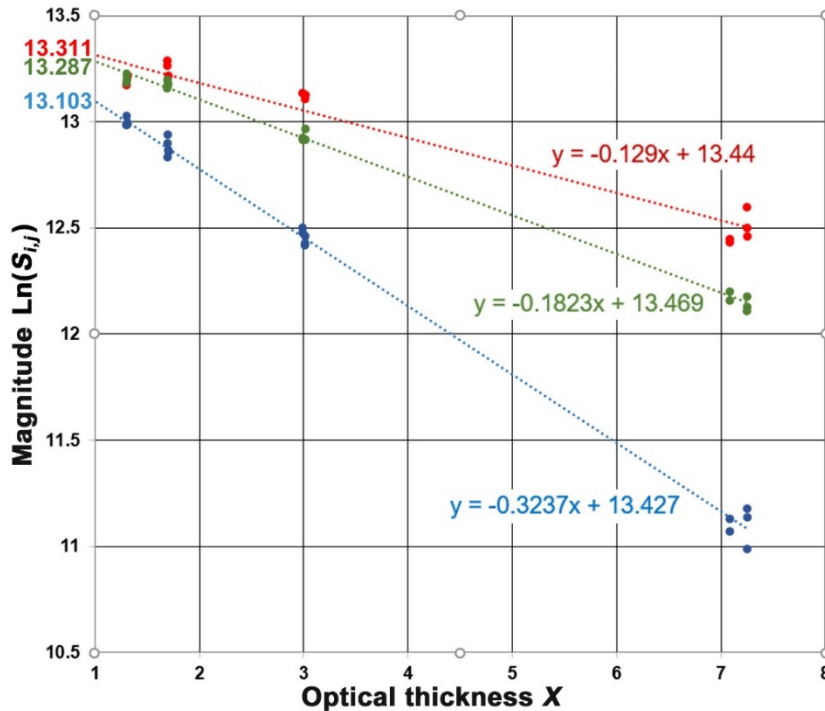


Figure 5. Graph of magnitude $\text{Ln}(S_{i,j})$ versus optical thickness X for Star #1 with trend lines extended to $X = 1$.

The stars in Table 1 are G2V stars so we will use them to extend the previous analysis to evaluate W_j . By substituting $x = 1$ in each of the trend line equations in Figure 4, we can solve for the corresponding y -value (13.311, 13.287 and 13.103 for the red, green and blue spectral bands, respectively). This is shown in **Figure 5**. These values represent the magnitude of the star in each of the three spectral bands were the star directly overhead ($X = 1$). Exponentiating these

values retrieves their corresponding raw signal values $S_{i,j}$ in ADUs: $S_{i,r} = 603801$, $S_{i,g} = 589482$, and $S_{i,b} = 490411$.

Since the atmospheric transmissivity factor $A_j = 1$ at $X = 1$, we can simplify Eq. 1 for stars directly overhead:

$$S'_{i,j} = S_{i,j} / (A_j W_j) = S_{i,j} / (1 \cdot W_j) = S_{i,j} / W_j \quad [\text{Eq. 4}]$$

Substituting our raw signal values into this equation, we get $S'_{i,r} = 603801/W_r$, $S'_{i,g} = 589482/W_g$, and $S'_{i,b} = 490411/W_b$. For G2V stars, our assumption is that the calibrated signals for the three spectral bands are equal, *i.e.*, $S'_{i,r} = S'_{i,g} = S'_{i,b}$. So, taking the ratios of the signals:

$$S'_{i,g} / S'_{i,r} = 1 = (589482 / W_g) / (603801 / W_r) = 0.9763(W_r / W_g) \quad [\text{Eq. 5}]$$

$$S'_{i,b} / S'_{i,r} = 1 = (490411 / W_b) / (603801 / W_r) = 0.8122(W_r / W_b) \quad [\text{Eq. 6}]$$

In these ratios, I've put the largest value of $S_{i,j}$ (in this case, the red spectral band) in the denominator. Re-arranging these equations, we get:

$$W_g = 0.9763W_r \quad \text{and} \quad W_b = 0.8122W_r \quad [\text{Eq. 7}]$$

As stated earlier, we're actually interested in the *relative* magnitudes of the weight factors. If we let $W_r = 1$, then $W_g = 0.9763$ and $W_b = 0.8122$. So, with a little math, we've determined the weight factors for Star #1. If we perform these calculations for all four stars in Table 1 and average the results, we get $W_r = 1$, $W_g = 0.9966$ and $W_b = 0.7976$.

Graphically, applying the weight factors for Star #1 to the data in Figure 5 wouldn't affect the slopes of the trend lines but would shift the lines for the green and blue spectral bands upward so all three would converge on a single point (magnitude 13.311 at $X=1$).

If you use the same imaging setup every time, then one set of W_j values should be okay for all your image processing. You'll still have to determine W_j one time but, after that, you should be able to use those values until you make a change in your equipment. For example, I switched from Astrodon to Baader RGB filters at the start of 2015. **Table 2** shows W_j values that I determined before and after the change. There is considerable difference between the two sets of weight factors, which is likely related to the filters' bandpass characteristics.

Table 2. Weight factors determined using RGB filters from two manufacturers.

| Manufacturer | W_r | W_g | W_b |
|--------------|--------|--------|--------|
| Astrodon | 0.9658 | 1 | 0.9071 |
| Baader | 1 | 0.9925 | 0.7933 |

Final thoughts

While W_j values determined for your imaging setup can be considered constants, for best results you should determine k_j values that are specific to each of your imaging sessions. This doesn't

mean that you have to shoot a separate set of images just for evaluating them. Recall that *stars used to determine k_j can be of any spectral class*. The images you capture of an astronomical object will probably contain some unsaturated field stars around the main target, as in **Figure 6**. If your imaging session lasts a good portion of the night, then the object should traverse enough sky for there to be a significant variation in stellar altitude. If not, just wait and shoot some additional images when the object gets close to the horizon. This should allow you to photometrically extract enough data from stars in your images to determine k_j values specific to your imaging session.

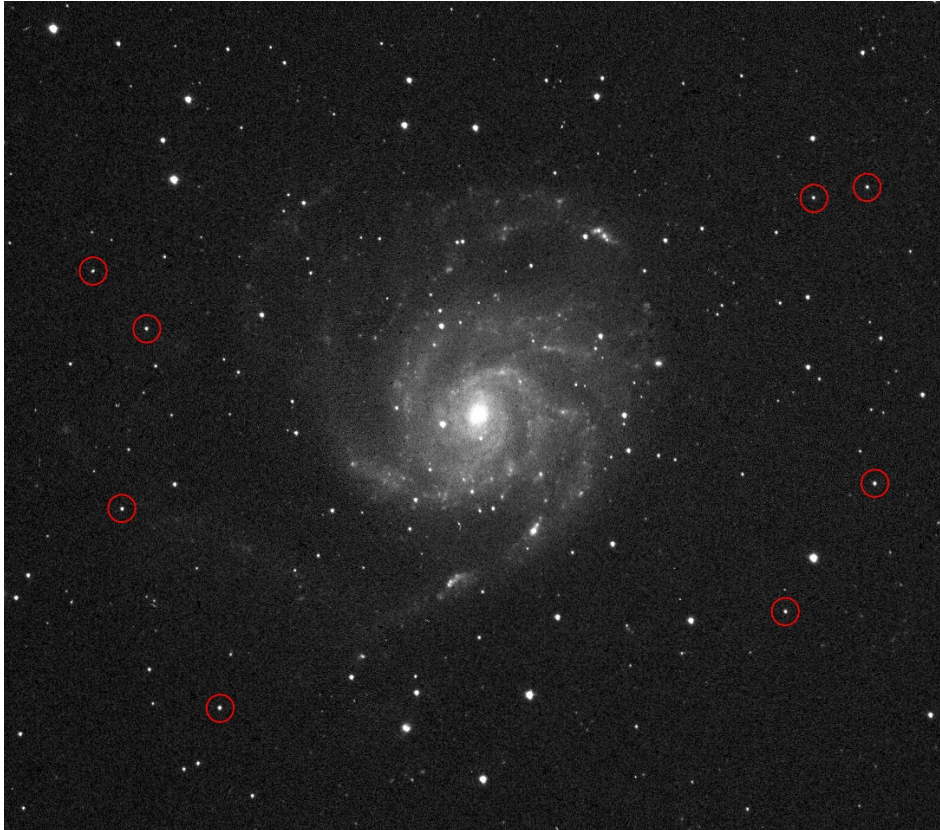


Figure 6. Raw image with field stars (circled) that could be used for evaluating k_j .

Okay, I know you're saying, "This looks like a lot of work." For astro-imagers like me who shoot monochrome images using red, green and blue filters and save them in FITS format, I've written a computer application that automatically analyzes your images to extract the data for the target stars, including calculating the zenith angles and raw signal values. It even plots the data *à la* Figure 4 and determines the extinction coefficients and weight factors. This application will run on any 32-bit or 64-bit Windows PC and can be downloaded for free at <http://cat-star.org/apps/apps-index.html>.

So, whether you use my application or calculate k_j and W_j manually, I hope what I've described will encourage you to get out and shoot target stars to determine your own values so you can objectively color calibrate your astro-imagery.